

Brane-Induced Gravity's Shocks

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(Dated: February 1, 2008)

We construct exact gravitational field solutions for a relativistic particle localized on a tensional brane in brane-induced gravity. They are a generalization of gravitational shock waves in 4D de Sitter space. We provide the metrics for both the normal branch and the self-inflating branch DGP braneworlds, and compare them to the 4D Einstein gravity solution and to the case when gravity resides only in the 5D bulk, without any brane-localized curvature terms. At short distances the wave profile looks the same as in four dimensions. The corrections appear only far from the source, where they differ from the long distance corrections in 4D de Sitter space. We also discover a new non-perturbative channel for energy emission into the bulk from the self-inflating branch, when gravity is modified at the de Sitter radius.

PACS numbers: 11.25.-w, 11.25.Mj, 98.80.Cq, 98.80.Qc

hep-th/0501028

In brane-induced gravity [1] gravitational force between masses on the brane should arise predominantly from the exchange of an unstable bulk graviton resonance, which falls apart at very large scales, instead of a stable zero-mode graviton governed by the usual four-dimensional (4D) General Relativity (GR). Far from sources the law of gravity is modified, and changes from 4D to a fully higher-dimensional one. Subtleties of gravitational perturbation theory in DGP have been explored in [2, 3, 4, 5] with the particular attention on the couplings of the longitudinal gravitons. It has been suggested that the brane extrinsic curvature may play the role of a coupling controller: the extrinsic curvature, responding to a source mass on the brane, may help to tame perturbation theory [2, 5]. Exploring this interplay between the curvature and the graviton couplings on exact solutions of DGP field equations could clarify the status of effective 4D theory. The DGP equations are, however, notoriously difficult to solve for localized sources [6].

In this Letter we present the first example of an exact solution for a localized particle in DGP models. We derive the gravitational field for a relativistic particle on a tensional brane in 5D. It generalizes 4D gravitational shock waves [7, 8, 9] in de Sitter space [10, 11, 12]. We present the closed form metrics for both the normal branch and the self-inflating branch DGP braneworlds [13]. The scalar graviton is absent in these metrics. One expects this in weakly-coupled perturbation theory, where the scalar graviton decouples from ultra-relativistic particles since to leading order its source vanishes, $T^\mu_\mu = 0$, and the corrections are suppressed by the source mass to momentum ratio $m/p \ll 1$ (e.g., hence the gravitational shock wave of 4D GR is also a solution in Brans-Dicke theory). Our results show that this persists in DGP, and that the dangerous strongly coupled mode found in perturbation theory does not destroy the solutions at large momenta after all. Further, we find that

as long as the brane-localized terms are present, at short distances the wave profile behaves exactly like in 4D GR [7, 8]. The deviations appear only far from the source, where they differ from the 4D GR corrections in de Sitter space. We also suggest a new way for checking graviton effective field theory with our shock waves.

DGP braneworlds [1] are given by a bulk action with metric kinetic terms in both the bulk and on the brane, with couplings set by bulk and brane Planck scales, $\kappa_4^2 = 1/M_4^2$ and $\kappa_5^2 = 1/M_5^3$. In 5D, the field equations are [1]

$$M_5^3 G_5^A{}_B + M_4^2 G_4^\mu{}_\nu \delta_\mu^A \delta_B^\nu \delta(w) = -T^\mu{}_\nu \delta_\mu^A \delta_B^\nu \delta(w), \quad (1)$$

where $G_5^A{}_B$, $G_4^\mu{}_\nu$ are bulk and 3-brane-localized Einstein tensors; $T^\mu{}_\nu$ is the brane stress-energy; $\{A, B\}$ and $\{\mu, \nu\}$ are bulk and 3-brane world-volume indices, respectively. We use Gaussian normal coordinates for the 5D metric, $ds_5^2 = dw^2 + g_{\mu\nu}(w, x) dx^\mu dx^\nu$, orbifolding by a Z_2 symmetry $w \rightarrow -w$ around the brane at $w = 0$. For a brane with tension $\lambda \neq 0$ and no matter, $T^\mu{}_\nu = -\lambda \delta^\mu{}_\nu$. The solution is a de Sitter 3-brane in a flat bulk [13],

$$ds_5^2 = (1 - \epsilon H|w|)^2 ds_{4dS}^2 + dw^2. \quad (2)$$

Apart from $\epsilon = \pm 1$ in (2), this is identical to the 5D version [14] of the inflating VIS domain wall [15]. We shall use $ds_{4dS}^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{(1 - H^2 r^2)} + r^2 d\Omega_2^2$, the static de Sitter metric instead of the spatially flat one employed in [14]. The space-time of (2) is a 4D de Sitter hyperboloid in a 5D Minkowski bulk. Here ϵ arises because we can retain either the interior $\epsilon = +1$ or the exterior $\epsilon = -1$ of the hyperboloid after orbifolding, thanks to the brane curvature. The junction conditions relate the 4D curvature and the brane tension as [13]

$$H^2 + \epsilon \frac{2M_5^3}{M_4^2} H = \frac{\lambda}{3M_4^2}. \quad (3)$$

The solutions with $H > 0$ differ for each of $\epsilon = \pm 1$ ($H < 0$ cases are their PT transforms). On the normal branch, $\epsilon = +1$, the solution reduces to the 4D Friedman equation in the limit $M_5 \ll M_4$, $3H^2 \simeq \lambda/M_4^2$. In the bulk, we

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keep the interior of the hyperboloid. It has finite volume and so the perturbative 4D graviton exists. The brane curvature terms $\sim M_4^2 \int d^4x \sqrt{g_4} R_4/2$ suppress the couplings of the $m_g > 0$ graviton KK modes, producing the 4D effective theory when $M_5 \ll M_4$. On the self-inflating branch with $\epsilon = -1$, at low tensions $\lambda \ll 12M_5^6/M_4^2$, the eq. (3) yields $H \sim 2M_5^3/M_4^2$. In the bulk we keep the exterior of the hyperboloid which has infinite bulk volume and so there is no perturbative 4D graviton. The effective theory arises only from the exchange of the bulk resonance.

Suppose now there is also a single relativistic particle, say a photon, with a momentum p on the brane, moving along some null geodesic of the brane-induced metric. Its stress-energy tensor sources an additional gravitational field. To find it, one could boost the linearized gravitational field for a massive particle by an infinite amount, simultaneously taking the limit $m \rightarrow 0$ such that $m \cosh \gamma = p$ stays finite [7, 10, 11, 16]. Viewing the metric as an expansion in the powers m , in the relativistic limit the terms of order higher than m vanish because there is only one factor of the boost parameter $\cosh \gamma$, and the linearized solution becomes exact! Alternatively, one can use a variant of a very elegant cut-and-paste technique developed by Dray and 't Hooft [8] for flat 4D backgrounds, and later applied to general 4D GR backgrounds by Sfetsos [12]. We will follow this route here, by extending the technique to DGP. First, we cast (2) in suitable null coordinates, defining $u = \frac{1}{H} \sqrt{\frac{1-Hr}{1+Hr}} \exp(Ht)$ and $v = \frac{1}{H} \sqrt{\frac{1-Hr}{1+Hr}} \exp(-Ht)$. We also change w to $|z| = -\frac{1}{\epsilon H} \ln(1 - \epsilon H|w|)$ (note that $\delta(w) = \delta(z)$), to simplify evaluating the curvature tensors with a conformal map as in [17]. The metric (2) becomes

$$ds_5^2 = e^{-2\epsilon H|z|} \left\{ \frac{4du dv}{(1 + H^2 uv)^2} + \left(\frac{1 - H^2 uv}{1 + H^2 uv} \right)^2 \frac{d\Omega_2}{H^2} + dz^2 \right\}. \quad (4)$$

Let the photon move along the v -axis on the brane, i.e. along the null geodesic $u = 0$. The observer on the North Pole of the brane de Sitter space sees a photon streaming away along the past horizon, starting from $r = 0$ at infinite past. Using the trick of Dray and 't Hooft to construct the photon's gravitational field, we introduce a jump in the v coordinate at $u = 0$ [8]: let $v \rightarrow v + \Theta(u)f$ and $dv \rightarrow dv + \Theta(u)df$ where f is the wave profile which depends only on the spatial transverse coordinates (in our case, the angles on the 2-sphere and z), and $\Theta(u)$ is just the usual step function. Changing the coordinates to $\hat{v} = v + \Theta(u)f$ yields $dv \rightarrow d\hat{v} - \delta(u)f du$. Substituting $v, dv \rightarrow \hat{v}, d\hat{v} - \delta(u)f du$ in (4) and dropping the carets gives

$$ds_5^2 = e^{-2\epsilon H|z|} \left\{ \frac{4du dv}{(1 + H^2 uv)^2} - \frac{4\delta(u)f du^2}{(1 + H^2 uv)^2} + \left(\frac{1 - H^2 uv}{1 + H^2 uv} \right)^2 \frac{d\Omega_2}{H^2} + dz^2 \right\}. \quad (5)$$

Substituting (5) into (1), including the photon source in

$T^\mu{}_\nu = -\lambda \delta^\mu{}_\nu + 2(p/\sqrt{g})g_{uv}\delta(\theta)\delta(\phi)\delta(u)\delta_v^\mu\delta_\nu^u$, and evaluating the curvature, bearing in mind that $\delta(u)$ and its derivatives are distributions (so $u\delta(u) = 0$, $u^2\delta^2(u) = 0$ and $f(u)\delta'(u) = -f'(u)\delta(u)$ [8, 12]), we find the independent field equations [18]. One is just eq. (3). The other comes from the $G_5{}_{uu}$ component of (1) and yields a *linear* field equation for the wave profile:

$$\frac{M_5^3}{M_4^2 H^2} \left(\partial_z^2 f - 3\epsilon H \partial_{|z|} f + H^2 (\Delta_2 f + 2f) \right) + (\Delta_2 f + 2f) \delta(z) = \frac{2p}{M_4^2} \delta(\Omega) \delta(z). \quad (6)$$

Here Δ_2 and $\delta(\Omega)$ are the Laplacian and the δ -function on a 2-sphere, peaked at $\theta = 0$. In the limit $M_5 \rightarrow 0$ the bulk derivatives disappear, one can factor out $\delta(z)$ and recover the 4D de Sitter equation of [10, 12].

It is simpler to solve eq. (6) for two sources with the same momentum p , running in the opposite directions in the static patch (4) [10, 11]. In this way we avoid some unphysical divergences when the last term in (7) vanishes for $l = 1$, which cancel out anyway by symmetry. The two-source solution correctly represents the limit of infinite boost of the Schwarzschild-de Sitter geometry [10, 11]. We add an extra term, $\frac{2p}{M_4^2} \delta(\Omega') \delta(z)$, on the RHS of (6) where $\delta(\Omega')$ is peaked at $\theta = \pi$. To return to a single source, we can take the solution and multiply it by $\Theta(\pi/2 - \theta)$ as in [12]. So one particle moves along $\theta = 0$ and the other along $\theta = \pi$. With this choice of trajectories we use the addition theorem for spherical harmonics to replace them with Legendre polynomials $P_l(\cos \theta)$ in the expansion for the wave profile: $f = \sum_{l=0}^{\infty} (f_l^{(+)}(z) P_l(\cos \theta) + f_l^{(-)}(z) P_l(-\cos \theta))$. Here $f_l^{(\pm)}(z)$ are the bulk wave functions; $f_l^{(+)}$ is sourced by the photon at $\theta = 0$ and $f_l^{(-)}$ by the photon at $\theta = \pi$. By orthogonality and completeness of Legendre polynomials, the modes $f_l^{(\pm)}(z)$ obey the same differential equation,

$$\partial_z^2 f_l - 3\epsilon H \partial_{|z|} f_l + H^2 (2 - l(l+1)) f_l = \frac{M_4^2 H^2}{M_5^3} \left(\frac{(2l+1)p}{2\pi M_4^2} - (2 - l(l+1)) f_l \right) \delta(z). \quad (7)$$

We interpret δ -function on the RHS by pillbox integration as a matching condition for the first derivatives of $f_l^{(\pm)}$ on the brane. The remaining boundary conditions come from requiring orbifold symmetry $f_l(-z) = f_l(z)$ and square integrability in the bulk, so that the solutions are localized on the brane. Since both $f_l^{(\pm)}$ solve the same boundary value problem, $f_l^{(+)} = f_l^{(-)} = f_l$. By (7), f_l 's are simple exponentials, and to be localized on the brane (i.e. square-integrable) they must be $\sim e^{-[2l+(1-3\epsilon)/2]H|z|}$. Because $P_l(-x) = (-1)^l P_l(x)$, the solution will be an expansion only in even-indexed polynomials $P_{2l}(\cos \theta)$, like in 4D [10, 12]. Solving (7) for the bulk wave functions f_l 's [18] yields the solution for two relativistic photons on the brane moving in opposite

directions:

$$f(\Omega, z) = -\frac{p}{2\pi M_4^2} \sum_{l=0}^{\infty} \frac{4l+1}{(2l-1+\frac{(1-\epsilon)\mathbf{g}}{2})(l+1+\frac{(1+\epsilon)\mathbf{g}}{4})} \times e^{-[2l+(1-3\epsilon)/2]H|z|} P_{2l}(\cos\theta), \quad (8)$$

where $\mathbf{g} = 2M_5^3/(M_4^2 H) = 1/(Hr_c)$ (see [1]). The solution can be checked by direct substitution into (6), (7). When $\mathbf{g} = 0$, at $z = 0$ this reproduces the 4D series solution of [10] for both $\epsilon = \pm 1$. We can bring the series (8) to a more compact form. Factorizing the coefficients of the expansion, and defining $\tau = e^{-H|z|}$ and $x = \cos\theta$,

$$f(\Omega, z) = -\frac{[3 - (1-\epsilon)\mathbf{g}]p}{(3+\epsilon\mathbf{g})\pi M_4^2} \sum_{l=0}^{\infty} \frac{\tau^{2l+(1-3\epsilon)/2}}{2l-1+\frac{(1-\epsilon)\mathbf{g}}{2}} P_{2l}(x) - \frac{[3 + (1+\epsilon)\mathbf{g}]p}{2(3+\epsilon\mathbf{g})\pi M_4^2} \sum_{l=0}^{\infty} \frac{\tau^{2l+(1-3\epsilon)/2}}{l+1+\frac{(1+\epsilon)\mathbf{g}}{4}} P_{2l}(x). \quad (9)$$

Note that while for $\epsilon = 1$ the series (9) is finite, for $\epsilon = -1$, the self-inflating branch solutions, it displays a spectacular behavior as $\mathbf{g} \rightarrow 1$, when the brane tension vanishes, where the $l = 0$ term of the first sum has a pole ($\mathbf{g} = 3$ is regular, as seen from (8)). A closer look [18] shows that in this limit the only finite $l = 0$ solution of (7) is the delocalized bulk mode, not present in (8). For $\mathbf{g} = 1$ gravity is modified at the scale equal to the cosmological horizon on the brane. Brane and bulk begin to “resonate”: in a time-dependent problem, as $\mathbf{g} \rightarrow 1$ slowly, the $l = 0$ mode would grow larger, and at some point it may begin to produce delocalized gravitons, taking energy from the self-inflating brane into the bulk. This indicates an onset of a dramatic new instability, which warrants closer investigation.

Now, recall the generating function for Legendre polynomials, $(1 - 2x\tau + \tau^2)^{-1/2} = \sum_{l=0}^{\infty} P_l(x)\tau^l$. Using $\lim_{\epsilon \rightarrow 0} \int_{\epsilon}^{\tau} d\vartheta \vartheta^{l+\beta} = \frac{\tau^{l+\beta+1}}{l+\beta+1} - \lim_{\epsilon \rightarrow 0} \int^{\epsilon} d\vartheta \vartheta^{l+\beta}$ to regulate divergences when $l+\beta \leq -1$, integrate over τ ; after straightforward manipulations eq. (9) becomes

$$f(\Omega, z) = \frac{p}{2\pi M_4^2} \frac{\tau^{\frac{1-3\epsilon}{2}}}{1 - \frac{1-3\epsilon}{4}\mathbf{g}} - \frac{p}{2(3+\epsilon\mathbf{g})\pi M_4^2} \times \int_0^{\tau} d\vartheta \left(\frac{1}{\sqrt{1-2x\vartheta+\vartheta^2}} + \frac{1}{\sqrt{1+2x\vartheta+\vartheta^2}} - 2 \right) \times \left(\frac{[3 - (1-\epsilon)\mathbf{g}]\vartheta^{\frac{\mathbf{g}(1-\epsilon)-4}{2}}}{\tau^{\frac{(\mathbf{g}-3)(1-\epsilon)}{2}}} + \frac{[3 + (1+\epsilon)\mathbf{g}]\vartheta^{\frac{\mathbf{g}(1+\epsilon)+2}{2}}}{\tau^{\frac{(\mathbf{g}+3)(1+\epsilon)}{2}}} \right). \quad (10)$$

Since (10) is a Green’s function in the transverse directions, it has short-distance singularities at $x = \pm 1$, but is well-behaved elsewhere. We will not attempt to evaluate it for the general case. Here we only look at the formula for f on the brane $z = 0$ (i.e. $\tau = 1$) at transverse distances \mathcal{R} well inside the cosmological horizon, $\mathcal{R} \ll H^{-1}$. Note that if we rewrite eq. (3) as $(1+\epsilon\mathbf{g})H^2 = \lambda/(3M_4^2)$ we find that on the normal branch \mathbf{g} can vary between zero (4D) and infinity (5D), with the “resonance” mentioned above at $\mathbf{g} = 1$. On the self-inflating branch, $\mathbf{g} \leq 1$

or $r_c \geq 1/H$, as long as $\lambda \geq 0$. In the limit $\mathbf{g} \rightarrow 1$ on the self-inflating brane the tension vanishes.

When $\mathbf{g} = 0$, (10) is identical to the 4D case on both branches, as remarked above. Indeed, the integrals give

$$f_{4D}(\Omega) = \frac{p}{2\pi M_4^2} \left(2 - x \ln \left[\frac{1+x}{1-x} \right] \right). \quad (11)$$

The metric transverse to the null particle on the brane is $ds_2^2|_{z=u=0} = d\Omega_2/H^2$ and so the proper “radial” distance is measured by the polar angle θ . For small angles, $\mathcal{R} \simeq \theta/H$, $x = 1 - H^2\mathcal{R}^2/2$, and (11) reduces precisely to the flat 4D solution [7, 8]: up to $\mathcal{O}(\mathcal{R}^2/H^{-2})$ corrections, and with the sign conventions of [10, 12],

$$f_{4D}(\Omega) = \frac{p}{\pi M_4^2} + \frac{p}{\pi M_4^2} \ln \left(\frac{\mathcal{R}}{2H^{-1}} \right). \quad (12)$$

The constant term in this equation can always be recovered in the 4D flat case by a diffeomorphism. The integrals for $\mathcal{O}(\mathbf{g})$ corrections contain terms like $\int d\zeta \ln(1 + b\zeta)/\zeta$ and cannot be written in closed form [19]. However, at short distances the singular $\mathcal{O}(\mathbf{g})$ powers in the integrand precisely cancel out. In the remainder, since the leading order singularity in (10) is logarithmic around $\mathbf{g} = 0$, the additional logarithms soften the corrections further, and render them finite as $\mathcal{R} \rightarrow 0$. Hence at short distances the solution looks exactly the same as in 4D.

In fact this persists for any finite value of \mathbf{g} . If we rewrite the integrals (10) using the Euler substitutions $\zeta = \sqrt{1 \mp 2x\vartheta + \vartheta^2} + \vartheta$ and consider the limit $1-x \ll 1$, we extract the leading singularity. It comes from the term $\sim -\frac{p}{\pi M_4^2} \int_1^{1+\sqrt{2(1-x)}} \frac{d\zeta}{\zeta-x} = \frac{p}{\pi M_4^2} \ln \sqrt{\frac{1-x}{2}} + \text{finite terms}$ for all values of \mathbf{g} and ϵ . Substituting $x = 1 - H^2\mathcal{R}^2/2$, we recover the leading logarithm in (12). The subleading corrections in general differ from the terms in the expansion of the 4D de Sitter solution (11) around the short distance limit in flat space (12). In that case one finds corrections to come as even powers of $H\mathcal{R}$. In DGP, however, the corrections in (13) will come as odd powers of $H\mathcal{R}$ as well, starting with the linear term, signalling the hidden fifth dimension. Thus in general we will have

$$f(\Omega) = \frac{p}{\pi M_4^2} \left((1 + a_1 H^2 \mathcal{R}^2 + \dots) \ln \left(\frac{\mathcal{R}}{2H^{-1}} \right) + \text{const} + b_1 H\mathcal{R} + b_2 H^2 \mathcal{R}^2 + \dots \right), \quad (13)$$

where the coefficients a_k, b_k are numbers that can be computed explicitly for given values of \mathbf{g} and ϵ .

To see how 5D gravity reemerges, on the normal branch we can take the limit $\mathbf{g} \rightarrow \infty$. Then the background reduces to the inflating brane in 5D Minkowski bulk [14]. Using (9) to find the contribution of the second sum, which is $\propto \sum_{l=0}^{\infty} P_{2l}(x) = \frac{1}{\sqrt{8(1-x)}} + \frac{1}{\sqrt{8(1+x)}}$, and deducing the contribution of the first sum from the integral (10), we can write down the solution f_{5D} in closed form

[18]. At short transverse distances $x = 1 - H^2 \mathcal{R}^2/2$ the leading order behavior of f_{5D} , using $M_4^2 g = 2M_5^3/H$ and ignoring a constant and higher powers of \mathcal{R} , is

$$f_{5D}(\Omega) = -\frac{p}{2\pi M_5^3 \mathcal{R}} + \frac{3pH}{4\pi M_5^3} \ln\left(\frac{\mathcal{R}}{2H^{-1}}\right). \quad (14)$$

Notice the relative sign difference between the two terms on the RHS of (14). This is necessary in order for both to yield an *attractive* force $\propto -\vec{\nabla} f$ on a test particle in the shock wave background. The first term is just the 5D shock wave solution of [9]. The second term comes from the residual 4D graviton zero mode which persists on the normal branch when $g \rightarrow \infty$ because the bulk volume remains finite. Its perturbative coupling is set by the effective 4D Planck scale $M_{4eff}^2 = 4M_5^3/3H$, consistent with (14). At all sub-horizon distances the inverse power of \mathcal{R} in (14) wins over the logarithm and so in this limit gravity really looks five-dimensional inside the cosmological horizon.

The shock wave solutions derived here give us a clear and explicit demonstration that the gravitational “filter” mechanism of [1] may in fact work beyond perturbation theory. The key is that the coefficients in (9) decrease with the index l of the Legendre polynomials. Since the momentum of a graviton mode on a transverse 2-sphere along de Sitter brane is proportional to the index of the polynomial, $q \sim Hl$, this means that the modes with $q > gH \simeq 1/r_c$ have amplitudes suppressed by q . This controls the rate of divergence of the series, and limits it to be no worse than a logarithm for any finite value of g . Hence at short distances $\mathcal{R} \leq r_c \simeq H^{-1}/g$, gravitational shock waves are exactly the same as in 4D GR. Only for very low momenta are the amplitudes unsuppressed, and so the shock wave profile changes towards 5D at large

distances $\mathcal{R} \geq H^{-1}/g$. We see this explicitly in the limit $g \rightarrow \infty$ on the normal branch: we recover 5D gravity since when $r_c \rightarrow 0$ all modes with finite momenta remain unsuppressed. Thus we expect that in DGP with finite g the leading order Planckian scattering behaves as in 4D GR, since the differences are suppressed by powers of m/p and $H\mathcal{R}$, both very small for highly energetic particles at short distances.

We note that our exact shock waves may be a new arena to explore dynamics of the scalar graviton in DGP. Imagine probing the gravitational field of an arbitrary mass on the brane with a very relativistic probe. Since local physics in DGP obeys the usual 4D diffeomorphism invariance, we can transform all of the relevant physics to the rest frame of the probe. In this frame, the source mass will appear to move with a very high speed v , and so its gravitational field should be well approximated by our shock wave solutions, with the corrections due to the mass suppressed by the powers of $m/p = \sqrt{1/v^2 - 1}$. One can then treat the rest mass of the source as a perturbation of the shock wave geometry, and study how the scalar graviton responds to it. If organized as an expansion in powers of m/p , perturbation theory may be under control. It would be interesting to investigate what happens with the strong coupling of [2, 3, 4, 5] in this case. This may shed new light on the interplay of background curvature and scalar graviton effective field theory.

Acknowledgements

We thank S. Dimopoulos, G. Dvali, R. Emparan, G. Gabadadze, M. Luty, K. Sfetsos and L. Sorbo for useful discussions, and the Aspen Center for Physics for hospitality. NK was supported in part by the DOE Grant DE-FG03-91ER40674, by the NSF Grant PHY-0332258 and by a RIA from the Research Corporation.

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